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construct the triangle $P_aP_bP_c$, whose sides are known, and then upon $b\sqrt{2}$ and $c\sqrt{2}$ construct isosceles right triangles. The vertices of these triangles (at the right angles) and the middle of the side $2c$ are the vertices of the required triangle.

Number of Solutions. In constructing $P_aP_bP_c$ we may take for its sides

$$\begin{array}{lll} a\sqrt{2}, & b\sqrt{2}, & 2c \dots (1). \\ \text{or } a\sqrt{2}, & 2b, & c\sqrt{2} \dots (2). \\ \text{or } 2a & b\sqrt{2}, & c\sqrt{2} \dots (3). \end{array}$$

In the special problem before us $a=1$, $b=2$, $c=2\frac{1}{2}$; whence

$$\begin{array}{lll} 1.414, & 2.828, & 5. \dots (1). \\ 1.414, & 4. & 3.535 \dots (2). \\ 2, & 2.828, & 3.535 \dots (3). \end{array}$$

In the first case there is no real solution.

In the second case P falls *without* the triangle, and $x=2\frac{7}{16}$.

In the third case P falls *within* the triangle, and $x=2\frac{1}{8}$.

The values are derived by scaling off from the figures. The construction here given is for the third case, the unit being one centimeter.

154. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O., University, Miss.

The angle between the edge of a trihedral angle and the bisector of the opposite face angle is less than, equal to, or greater than half the sum of the other two face angles, according as it is itself acute, right, or obtuse.

Solution by the PROPOSER.

Let $S-ABC$ be a trihedral angle and SD the bisector of the face angle ASB .

CASE I. $\angle DSC < \frac{1}{2}$ a right angle. (See Fig. 1.)

To prove $\angle DSC < \frac{1}{2}(\angle ASC + \angle BSC)$.

From C , any point of the edge SC , draw CD perpendicular to SD .

Through D , in the face ASB , draw AB perpendicular to SD . Then, SD is perpendicular to the plane ABC .

Connect S with F and E .

Comparing the right triangles AFD and BED , $AD=BD$ (since triangle ASD =triangle BSD), and the vertical angles at D are equal. Hence, the triangles are equal, and $DE=DF$.

It follows that right triangles FSD and ESD are equal, and $\angle FSD = \angle ESD$.

Now, since SD is perpendicular to the plane ABC , the plane DSC is perpendicular to the plane ABC . Therefore BE and AF , which lie in one of these planes and are perpendicular to their intersection, are perpendicular to the other plane, DSC .

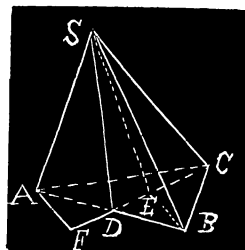


Fig. 1.

Consequently, SE and SF are the projections of SB and SA , respectively, on the plane DSC .

Therefore $\angle ESC < \angle BSC$, and $\angle FSC < \angle ASC$; or $\angle DSC - \angle DSE < \angle BSC$, and $\angle DSC + \angle DSF < \angle ASC$.

Adding, and remembering that $\angle DSE = \angle DSF$, $2\angle DSC < \angle BSC + \angle ASC$, $\angle DSC < \frac{1}{2}(\angle BSC + \angle ASC)$.

CASE II. $\angle DSC =$ a right angle. (See Fig. 2.)

To prove $\angle DSC = \frac{1}{2}(\angle ASC + \angle BSC)$.

Draw DX parallel to SC , and AB perpendicular to SD .

Then SD is perpendicular to DX , and, hence, to the plane determined by AB and DX .

This plane intersects the planes of faces BSC and ASC in BE and AF , respectively.

Since SC is parallel to DX , it is parallel to the plane BDX , and, hence, parallel to BE and AF .

Through SD pass a plane perpendicular to SC , intersecting the plane of AB and DX in EF , and the planes of faces BSC and ASC in SE and SF , respectively. Since CS is perpendicular to the plane FSE , so are BE and AF .

Hence, \angle 's BES , BED , AFS , and AFD are right angles.

Right triangles DAF and DBE are equal, since $AD = BD$ (from equality of right triangles ASD and BSD), and the vertical angles at D are equal.

Therefore $BE = AF$. Hence, since $SB = SA$, right triangles SEB and SFA are equal, and $\angle ASF (= \angle ASH) = \angle BSE$.

Now, since \angle 's CSD , CSE , and CSH are right angles, $\angle CSD = \frac{1}{2}(\angle CSE + \angle SCH)$, $= \frac{1}{2}(\angle CSB + \angle BSE + \angle CSA - \angle ASH)$ $= \frac{1}{2}(\angle CSB + \angle CSA)$.

CASE III. $\angle DSC >$ a right angle. (See Fig. 3.)

To prove $\angle DSC > \frac{1}{2}(\angle CSA + \angle CSB)$.

Produce AS , BS , and DS , forming another trihedral angle $S-A'B'C$.

By Case I, $\angle CSD' < \frac{1}{2}(\angle CSA' + \angle CSB')$, or $180^\circ - \angle CSD < \frac{1}{2}(180^\circ - \angle CSA + 180^\circ - \angle CSB)$, from which $-\angle CSD < -\frac{1}{2}(\angle CSA + \angle CSB)$.

Therefore, $\angle CSD > \frac{1}{2}(\angle CSA + \angle CSB)$.

Also solved by H. C. WHITAKER.

155. Proposed by J. C. NAGLE, M.A., C.E., Professor of Civil Engineering, State Agricultural and Mechanical College, College Station, Texas.

A special case of the following problem was sent me some time ago by an ex-member of one of my engineering classes, as occurring on the Southern Pacific Ry., near Devil's River:

Two straight tracks, p feet between centers, are to be united by a cross-over composed of two curves of radius R , and a length L of intervening tangent. Required the central angles and the distance between tangent points, measured along main track. In the special case referred to p was 62 feet, L 100 feet with $9^\circ 30'$ curves.

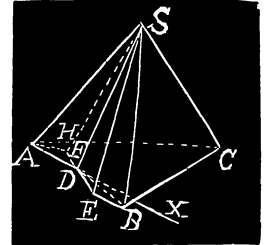


Fig. 2.

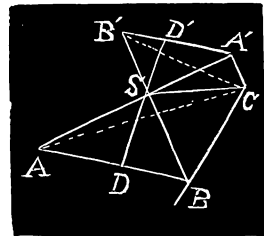


Fig. 3.